

# CLOSED TIMELIKE CURVES IN RELATIVISTIC COMPUTATION

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**ABSTRACT.** In this paper, we investigate the possibility of using closed timelike curves (CTCs) in relativistic hypercomputation. We introduce a wormhole based hypercomputation scenario which is free from the common worries, such as the blueshift problem. We also discuss the physical reasonability of our scenario, and why we cannot simply ignore the possibility of the existence of spacetimes containing CTCs.

## 1. INTRODUCTION

Gödel in 1949 introduced a cosmological solution of Einstein's field equations in which time travel is possible since it contains closed timelike curves (CTCs), see [17].<sup>1</sup> Besides Gödel's rotating universe, there are several other interesting and physically relevant spacetimes containing CTCs, such as Kerr-Newman rotating black holes [32, Prop.2.4.7] or Tipler's rotating cylinder [38] to mention only a few. For more physically realistic spacetimes containing CTCs, see, e.g., [9].

There are several papers using CTCs to design computers with higher computational power. For example, Brun [11] uses Novikov's principle of self-consistency, see, e.g., [16, p.1916], to design algorithms solving the prime factorization, NP-complete and even PSPACE-complete problems efficiently. There are other papers using Deutsch's causal consistency model [12] based on Everett's many world interpretation of quantum mechanics to prove theorems about CTC based quantum computation [1], [8].

These papers do not aim to challenge the physical Church-Turing thesis by using the CTCs for hypercomputation (i.e., physical computational scenarios which are able to solve non-Turing computable problems).

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*Key words and phrases.* closed timelike curves, hypercomputation, relativistic computation, Malament-Hogarth spacetimes, wormholes.

<sup>1</sup>Gödel's spacetime is not the first in the literature that turned out to contain CTCs.

In the literature, the well-known concept of Malament-Hogarth spacetime (MH-spacetime for short) is designed to capture those spacetimes which are suitable for hypercomputation, see, e.g., [13, §4.3], [14, §3], [18], [19]. A spacetime is called a MH-spacetime if there is an event (MH-event) whose causal past contains an infinitely long timelike curve. The idea behind this definition is that a computer traveling along this infinite path can compute forever and send a signal at any time during its computation which reaches an observer (programmer) before the MH-event.

Ignoring the potentially infinite space that the computer might require to carry out an infinite computing task, we can argue that hypercomputation can be implemented in any MH-spacetime as follows. Let us take, for example, the non-Turing computable problem of the decision of the consistency of Zermelo-Fraenkel set theory (ZF). While the programmer moves along a finite worldline through the MH-event, the computer checks all the proofs from the axioms of ZF one by one looking for a contradiction. If the computer finds a contradiction, let it send a signal to the programmer. It is easy to see that the programmer does not get any signal if and only if no contradiction can be derived from ZF. So if there is no signal before the MH-event then the programmer learns that ZF is consistent.<sup>2</sup>

It is easy to show that every spacetime containing a CTC is a MH-spacetime, see, e.g., [24, Prop.1]. So if the definition of MH-spacetimes were our only criteria to accept a spacetime suitable for hypercomputation, then here we could easily close our investigation with the conclusion that hypercomputation can be implemented in any spacetime containing CTCs.

Nevertheless, we do not stop here. In this paper, we introduce a scenario that uses wormhole based CTCs for hypercomputation concerning other aspects of spacetime needed for hypercomputation, such as the potentially infinite space required by the computation. We will show that our construction has many advantages over the hypercomputational scenarios of the literature, e.g., it is free from the common worries, such as the blueshift problem.

There are several interesting papers dealing with the connection of computation and the possibility of time travel. For example, Akl [2]

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<sup>2</sup>Let us note here that there is no contradiction with the fact that Gödel's second incompleteness theorem implies that the consistency of ZF cannot be derived from the axioms of ZF. This is so because, if ZF is consistent, the above hypercomputational scenario does not prove the consistency by deriving it, but decides the question of consistency by a physical experiment.

investigates the issue of non-universality of CTC based hypercomputation and Stannett [35] investigates the possibility of  $P \neq NP$  in spacetimes containing special kinds of CTCs.

For a survey on several physical and mathematical models related to hypercomputation, see, e.g., Stannett [34].

## 2. WHY DON'T WE SIMPLY BAN THE EXISTENCE OF CTCs?

Some physicists argue that time travel is too fancy to consider spacetimes containing CTCs physically reasonable. They usually suggest excluding them by assuming (as an axiom) that there are no CTCs in physically reasonable spacetimes. However, this direct way of banning CTCs has a serious drawback. Namely, if we would like to understand whether CTCs can or cannot occur in a “physically reasonable” spacetime (e.g., in our universe), it is important not to exclude these spacetimes by brute force (i.e., assuming that CTCs do not exist). As Monroe writes in his paper [25]: “Thus, causality assumptions, like Euclid’s parallel postulate, risk closing off interesting lines of investigation.” In general, if we ban a physical phenomenon by an axiom directly, we will not be able to investigate it any more. Therefore, we will never be able to (meaningfully) answer the question why this phenomenon cannot occur. So if we assume that CTCs cannot exist, the only thing we can say about why it is impossible to travel back to the past is because we have just assumed it.<sup>3</sup> In other words, if we simply ban CTCs by an axiom we will never get any clue why they cannot occur in our universe or whether they really cannot.

It is interesting that finding natural axioms defining the physically reasonable spacetimes is not an easy task at all. For example, the so called energy conditions, which were a kind of natural way to exclude the “undesirable” spacetimes, have come to seem less natural in recent decades. Among other things, this is so because some of them simply exclude the possibility of the accelerating expansion of our universe (which has been discovered in 1998, see, e.g., [33]). So for example, we cannot assume the “natural” strong energy condition if we want to model our actual universe within general relativity. So the strong energy condition is simply “dead,” but the weaker conditions are also “moribund” for various reasons, see, e.g., [10]. The condition of being “hole-free,” introduced by Geroch, also turned out to be too strong since Krasnikov has shown that even Minkowski spacetime does not satisfy it [21].

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<sup>3</sup>For more on why-type question in physics, see [36].

Spacetime theories consistent with CTCs are not only interesting because they can be used to design hypercomputers, but because the concept of time travel (independently of whether it is possible in our world or not) is interesting by itself from the point of view of logic since just like the Liar paradox it contains a kind of self-reference (which is the basis of the grandfather paradox as well as the other paradoxes of time travel).

### 3. PROBLEMS WITH PUTTING THE COMPUTER TO THE CTC REGION

In using CTCs for hypercomputation, it is a natural idea sending the computer back in time. However, if we put the computer exactly on the CTC, the computer will go through the same chain of events. So we have not created a hypercomputer. We have only created a time traveling computer trapped in an infinite loop.

To avoid this kind of infinite loop of events, we can try to send the computer back to a location close to its past self. However, if the computer calculates long enough, there will be a (potentially) infinite heap of computers which might turn into a black hole ruining the whole project of hypercomputation. For example, if ZF is consistent, a computer trying to decide its consistency will compute forever. Hence, in this case, our heap of computers will be infinite.

To avoid creating a black hole, we have to send the computer back such that the overall mass of the created computer heap does not increase too fast with respect to the radius of the heap. We have to avoid that the radius of the heap becomes smaller than the Schwarzschild radius corresponding to the overall mass of the heap. The Schwarzschild radius is proportional to the mass. So we can avoid creating a black hole if we ensure that the mass of the heap of computers is proportional to the radius of the heap. If the size of the computer does not increase, this task can be solved, e.g., by putting the spacelike separated occurrences of the computer on a line with fixed distances apart.

Let us now create a four dimensional<sup>4</sup> (toy) spacetime in which the scenario above can be implemented.

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<sup>4</sup>All the examples of this paper can be constructed in any spacetime dimension grater than 2, and some of them, e.g., Example 1, can even be constructed in two dimension.

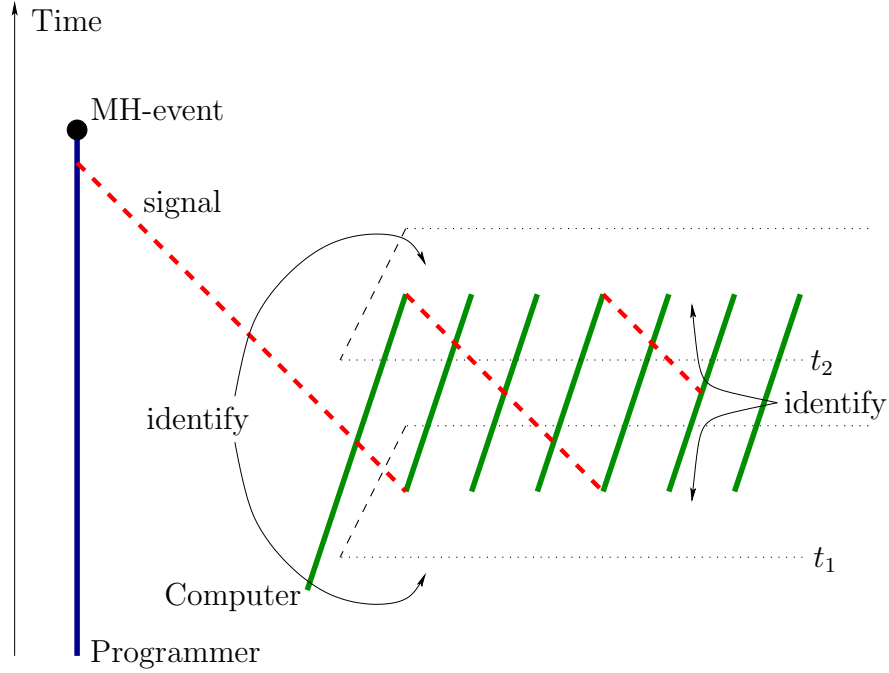


FIGURE 1. A 3 dimensional illustration of Example 1

**Example 1.** Let us identify the lower and the upper parts of two horizontal half hyperplanes, illustrated by two half spaces, in Minkowski spacetime, see Figure 1.<sup>5</sup>

It is easy to see that our CTC based hypercomputation scenario can be implemented in the spacetime of Example 1 if the size of the computer is fixed. However, fixing the size of the computer bounds its computing power, too. So because of this bound, we have only created a fast computer with limited computational power, but that is not a hypercomputer.

<sup>5</sup>By this identification, we do not change the metric of the Minkowski spacetime just its topology. We make the identification in the following way. Let us first choose two horizontal half hyperplanes such that they intersect the same vertical lines. Let us first remove the edges of these half hyperplanes (these points will not be part of our new spacetime). Let  $t_1 < t_2$  be the time coordinates corresponding to the two half hyperplanes. Let us connect the edges of the interval  $[t_1, t_2]$ ; and similarly let us connect the edges of the half lines  $(\infty, t_1)$  and  $[t_2, \infty)$  in every vertical line intersecting the interior of the two half hyperplanes, i.e., every vertical line intersecting the interior of the half hyperplanes is replaced with a line and a circle. The circles merge into a 4 dimensional half cylinder which will be the CTC region of the spacetime and the lines merge into a Minkowski half space. These two new regions is connected through the other Minkowski half space, which we have left unchanged.

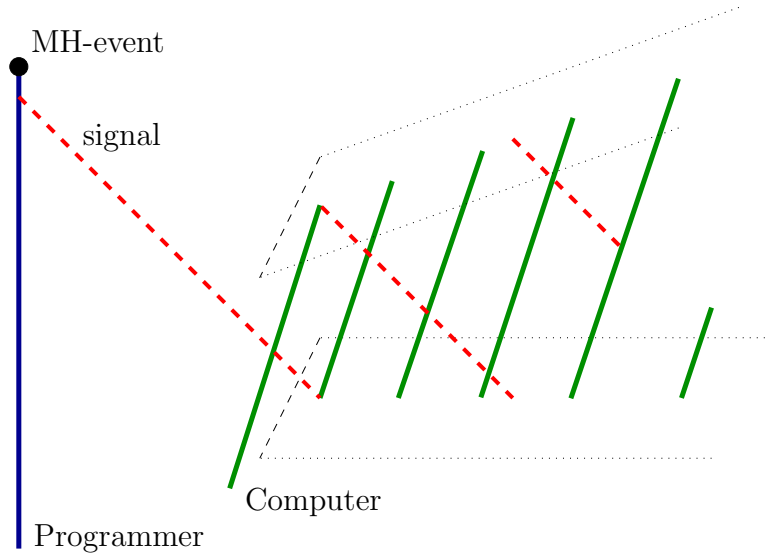


FIGURE 2. A 3 dimensional illustration of Example 2

It is highly probable that the MH-spacetime of Example 1 does not contain enough space for hypercomputation. This is so because the distance of the spacelike separated instances of the computers can only be increased by increasing the velocity of the computer which is bounded from above by the speed of light.

We can overcome the distance problem of Example 1, by changing it the following way.

**Example 2.** Let us identify, by the method used in Example 1, two non parallel spacelike half hyperplanes such that the distance of the spacelike separated instances of the computer increases without changing its speed, see Figure 2.

In Example 2, we have given a CTC based MH-spacetime in which the (slow enough) increase of the mass of the computer does not risk that the heap of computers will turn into a black hole.

If the size of the computer increases with time, we have to send it back such that the distance of the spacelike separated instances of the computer increases, too. By Examples 1 and 2, we have illustrated that whether this can or cannot be done depends on the type of the CTC region we use.

It is clear that sending the computer back to the past over and over again exaggerated the difficulty of increasing the size of the computer without creating a black hole. This is so because we not only have to be careful not to turn the computer itself into a black hole, but we also

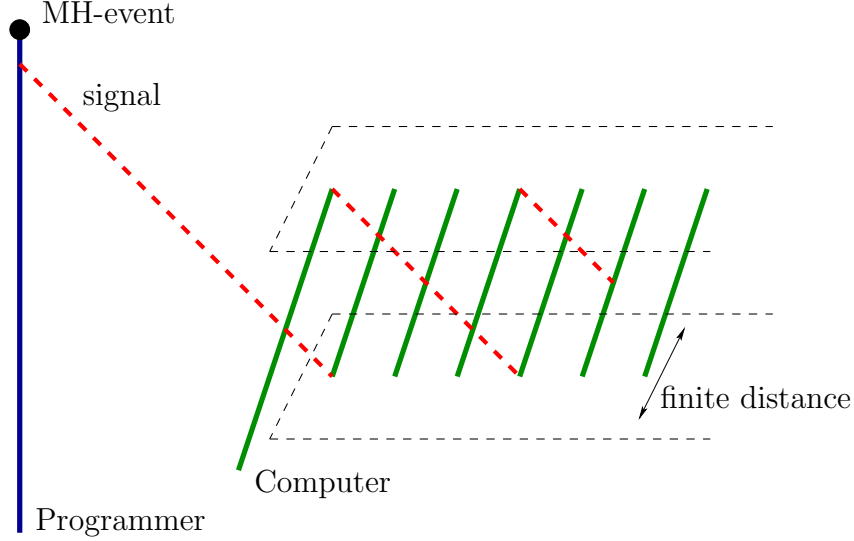


FIGURE 3. A 3 dimensional illustration of Example 3

have to ensure that the distances between its simultaneous appearances increase fast enough. Without sending it back to the past, we could easily ensure that the computer will never turn into a black hole by increasing its size faster than its mass.

By the arguments above, it is likely that the computer does not have enough space for hypercomputation in the spacetime of Example 1. However, except if the spacetime is 2-dimensional, our argument does not prove undoubtedly that the computer, by some cunning trick, cannot use the infinite space orthogonal to its movement without turning into a black hole.

So let us construct a (4-dimensional) MH-spacetime in which the computer clearly does not have infinite space for hypercomputation.

**Example 3.** Let us replace the half-spaces with 3-dimensional stripes having finite width and identify them the same way as in Example 1.

Example 3 is clearly a MH-spacetime in which hypercomputation is not possible due to the lack of space. Our Example 3, shows the existence of MH-spacetimes, in which hypercomputation cannot be implemented. So (without regarding the question of physical relevance) there are MH-spacetimes in which the project of hypercomputation cannot be carried through. Therefore, the concept of MH-spacetimes has to be refined if we want to capture the concept of those (not necessarily physically reasonable) spacetimes in which hypercomputation is possible.

Several papers, e.g., Etesi-Németi [15] or Németi-Dávid [29] put great effort into showing that in appropriate cosmological backgrounds (which are consistent with our experimental data about our real universe) the Kerr-Newman rotating black holes are physically relevant spacetimes suitable for hypercomputation. For example, [29] deals not only with the issue of the potentially infinite space the computation requires or the famous blueshift problem, but it also takes into account the relevant quantum theoretical considerations, such as the evaporation problem of black holes due to Hawking radiation. However, these papers do not try to introduce precise definitions to extend the concept of MH-spacetimes to capture the spacetimes which are suitable for relativistic hypercomputation.

Manchak [24] introduces some explicitly defined properties (such as, signal reliability condition or finite acceleration condition) of MH-spacetimes to make them more realistic for hypercomputation. However, he does not list any property ensuring the potentially infinite space required by the computation in the desirable properties. Moreover, his construction does not contain infinite space for the computer to compute.

Obviously, relativistic hypercomputation is an infinitely expensive project. So at first it may sound strange trying to lower its cost. However, even if it is infinitely expensive, it is not the same if the maintenance of the computer costs \$1 per century or \$1,000,000 per minute. Let us note here that from the point of view of the hypercomputation project the computer can be arbitrarily slow since it has infinite time to compute. So if the computer travels along a geodesic, its yearly maintenance cost can be quite cheap if it calculates slow enough.

According to a conjecture of Andréka-Németi-Wütrich, in every spacetime where the CTCs are created by some kind of rotation, the CTCs have to counter rotate with the rotation creating them. This conjecture is valid in all the well-known spacetimes where CTCs are created by rotation [7].<sup>6</sup> To counter rotate with a rotating mass the computer has to be accelerated.<sup>7</sup> As the mass (size of its data storage) of the computer increases during its calculation it becomes more and more expensive (it takes more and more energy) to accelerate it. And that can make it difficult (if not impossible) to keep the yearly cost (required energy) of the computer bounded above.

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<sup>6</sup>For a visual explanation of this counter rotation effect in the case of Gödel's universe, see [30, §6].

<sup>7</sup>For example, Malament showed that the total integrated acceleration of any CTC in Gödel spacetime is at least  $\ln(2 + \sqrt{5})$  [23].



#### 4. A CTC BASED RELATIVISTIC HYPERCOMPUTER THAT ACTUALLY WORKS WITHOUT PROBLEMS

Here we are going to present a relativistic hypercomputational scenario based on CTCs that works without the problems presented in Section 3. All the problems above were generated by sending the computer back in time. So let's try to design a hypercomputer using CTCs without sending the computer back to the past.

Is it possible to exploit the CTCs without using it for sending the computer back to the past? Yes, it is possible. The key idea is to send only the final result of the computation back to the past.

Let us first create, by using the “cut and paste” method of our previous examples, a spacetime containing a CTC in which hypercomputation is possible without sending the computer back to the past.

**Example 4.** Let us identify, by the method used in Example 1, the sides of two “stationary” boxes in the Minkowski spacetime such that identification of one of them begins (much) earlier than the other, see Figure 4.

By Example 4, we have created a kind of wormhole such that one of its mouths is in the past relative to its other mouth.

Let us now see how a hypercomputational scenario (e.g., decision of consistency of ZF) can be implemented in this spacetime. It is clear that, in this spacetime, the computer has enough space and time to compute remaining outside the CTC region.

Let the computer send a signal to mouth A of the wormhole if it derives a contradiction from ZF. By the identification, the signal comes out from mouth B earlier. Now let some device send (reflect) the signal back to mouth A. If the time delay between mouths A and B is more than the time that the signal has to take to reach mouth A from mouth B, the signal enters earlier to mouth A. Repeating this cycle the signal comes out from mouth B in the far past (only a little later than the initialization of the computer) where the programmer waits for the information. So if the computer derives a contradiction from ZF the programmer receives the signal. And if ZF is consistent (i.e., no contradiction can be derived from it), the programmer does not receive any signal and learns that ZF is consistent.

It is an important feature of the construction that the computer can send a signal back to the past at any time during its infinite computation. It is clear that not every spacetime containing a CTC can be used to implement this scenario. For example, the spacetimes of Examples

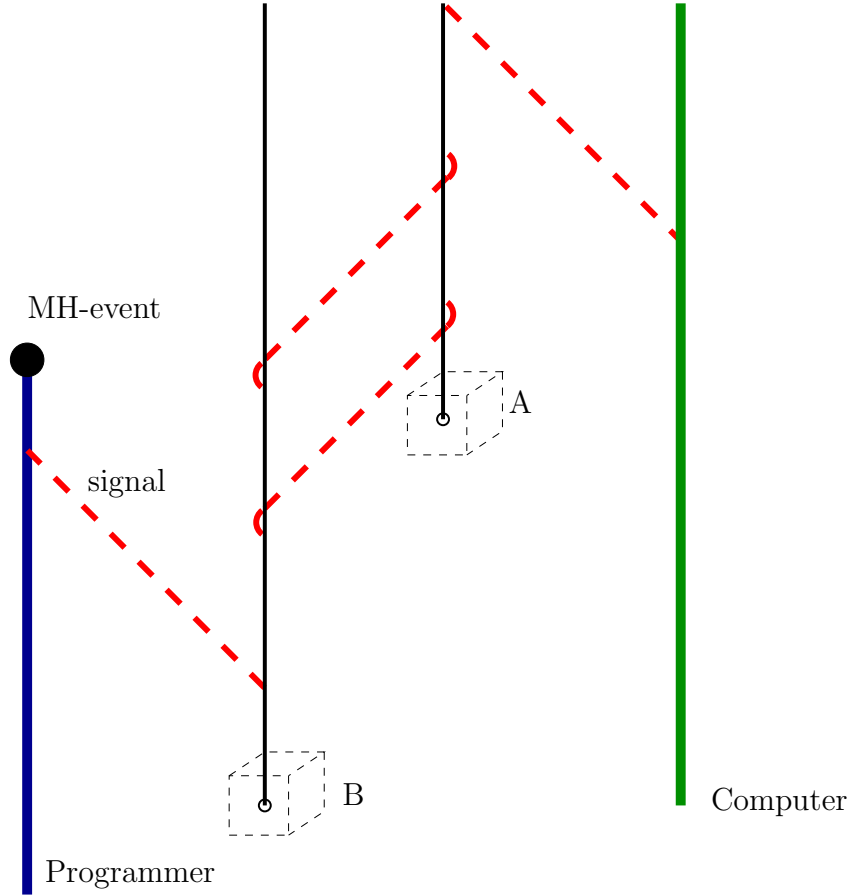


FIGURE 4. Illustration of Example 4

1, 2 and 3 are not suitable for a computer avoiding the CTC region to send signals back to the past.

It is clear that in our scenario, based on Example 4, sending only the final signal back to the past using the CTC region eliminates all the problems of Section 3. Moreover, the famous blueshift problem (see, e.g., [15], [29]) of hypercomputation based on Kerr-Newman black holes simply does not show up in this scenario.

##### 5. PHYSICALLY MORE REALISTIC SPACETIMES USING CTCs FOR HYPERCOMPUTATION

In Example 4, we have used the “cut and paste” method for creating our spacetime, which is a good method for creating counter examples to show that some logical implications about spacetimes do not hold. However, this method does not have any other physical relevance.

By using the twin paradox theorem of relativity theory, we can slow-down time at one of the mouths of the wormhole, see, e.g., [16, §B], [27, §I], [28]. This slow-down creates a spacetime similar to our spacetime of Example 4 in which the above hypercomputational scenario can be realized the same way. So we can create a CTC region similar to that of our “cut and paste” toy model above by using the twin paradox and wormholes.

**Example 5.** We take a spacelike wormhole and accelerate one<sup>8</sup> of its mouths in such a way that after the acceleration the time dilation, caused by the twin paradox effect, between the two mouths is grater than the amount of time a signal needs to travel from one mouth to the other, see Fig. 5.

In the construction of our wormhole based spacetime, we have only used the twin paradox theorem of relativity theory and wormholes, whose existence is consistent with the theory of general relativity. Moreover, wormholes have become more realistic after the discovery of the acceleration of the expansion of our universe, see, e.g., [10].

## 6. ON THE PHYSICAL REASONABILITY OF THIS SETUP

The rotating black hole based hypercomputation sends the programmer into a black hole and leaves the computer outside to compute forever, see, e.g., [15], [29]. A nice feature of our wormhole based hypercomputation as opposed to the rotating black-hole scenarios is that the programmer can remain at home and does not have to travel into a rotating black hole. A drawback is that we do not know whether there are wormholes in our universe or how to create them. While there is strong observational evidence of the existence of huge rotating black holes (see, e.g., [26]) which are ideal for hypercomputation, we do not have such strong experimental evidence for the existence of wormholes. However, there are attempts to detect wormholes, and some astronomical objects seem promising, see, e.g., [20]. Of course, if there are no (non-quantum size) wormholes in our universe, there is still hope that one day we will be able to create some, e.g., by enlarging some quantum wormholes [27, §H], [28] or by using the Casimir effect [40]. However, these are only speculations right now.

So wormholes are only speculative objects for the time being. To put some optimism to the end of this section, let us close it by the following claim of Visser written in 1997: “The good news about Lorentzian

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<sup>8</sup>The mouth of the wormhole can be accelerated, e.g., by accelerating some ordinary matter before the mouth which will dragging it by gravitation, see [31, p.146].

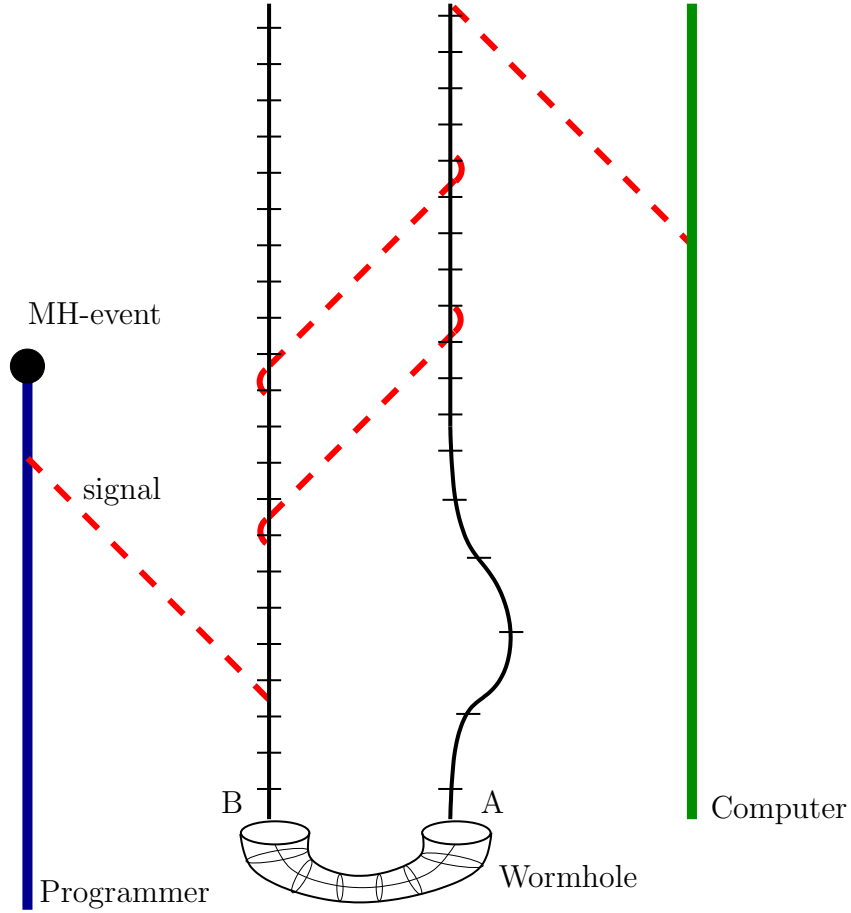


FIGURE 5. Illustration of Example 5

wormholes is that after about ten years of hard work we cannot prove that they don't exist." [40].<sup>9</sup>

## 7. REPEATABILITY OF THE HYPERCOMPUTATIONAL EXPERIMENT

So we might decide whether ZF set theory is consistent or not via our hypercomputer. If the answer is yes, probably we would like to use another hypercomputer to check whether a stronger axiom system (e.g., ZF together with some large cardinality assumption) is still consistent. If we found out that ZF is inconsistent, obviously we would like to use a hypercomputer to check the consistency of some weaker

<sup>9</sup>That is, so far there is no observation of a wormhole (or laboratory experiment providing one), but at least it is difficult to disprove their existence based on our spacetime theories.

set theory (e.g., the theory of hereditary finite sets). Not to mention that there are other interesting non-Turing computable questions besides the decision of the consistency of set theories. So it would be nice if the hypercomputational experiment would be repeatable.

The hypercomputational scenarios are typically not repeatable or their repeatability strongly depends on things that we do not know or cannot influence, such as what the programmer finds inside the rotating black hole he has jumped into.

What about the repeatability of our wormhole based hypercomputation? If there are wormholes, then probably there are more than one or if we can create wormholes, then probably we can create more than only one of them. So, if we can have one, it is plausible that we can have several from the key part of our hypercomputer. What about the infinite spacetime that the hypercomputer consumes during its computation? It is reasonable to assume that our universe is potentially infinite (since its expansion accelerates right now and we have absolutely no reason to think that this fact will change in the future). If we have infinite space for the first computer, then we will have enough space for the second, the third, etc. computers if we use it wisely. The trick is simple: use only  $1/2,000,000$  of the infinite space for the first computer (it is enough since it is still infinite) use  $1/4,000,000$  for the second computer,  $1/8,000,000$  for the third, etc. Then we will never pollute more than  $1/1,000,000$  of our universe with the electronic waste of our hypercomputers. So it is reasonable that the wormhole based hypercomputation is repeatable.

## 8. FASTER THAN LIGHT MOTION AND CLOSED TIMELIKE CURVES

It is a common belief that faster than light motion, which is possible in (1+1) dimension even for observers, see, e.g., [3, §2.7], [22, §2.7], entails CTCs. In the case of (1+3) dimension, the fact that no observer can move faster than light can be derived (in the sense of mathematical logic) from a streamlined axiom system of special relativity called **SpceRel**, see [5, Theorem 2.1]. In the proof of Theorem 2.1 of [5], it was strongly used that the dimension of space is greater than that of time. This fact and that there can be observers moving faster than light in the case of (1+1) dimension motivates us to conjecture that there is a natural generalization of **SpecRel** for the case of (3+3) dimension in which faster than light motion is allowed for observers.

If an observer sends out a faster than light signal, this signal travels backwards in time according to some observers moving relative to him. This fact suggests that, if observers can move faster than light, then

they can travel back in time. So the above common belief about CTCs and faster than light motion seems reasonable.

However, it is not true. Using the axiomatic method it is possible to show that faster than light motion of observers does not imply the possibility of time travel by itself. Moreover, time travel is possible by using faster than light observers only if it is possible without them.<sup>10</sup>

Why are we interested in  $(1+1)$ -dimensional and  $(3+3)$ -dimensional spacetime theories when we are apparently living in a  $(1+3)$ -dimensional one? Because we are logicians and therefore we are interested in logical connections of the possible axioms (basic assumptions) of our theories including the ones concerning the space and time dimensions. Besides this, we also would like to understand all the possible universes (that could have been created) and not just our sole universe we live in.

## 9. CONCLUDING REMARKS

We have seen that sending a computer back in time is a kind of awkward and problematic way to use CTCs to create hypercomputers. The idea of sending only the final result of the computer back in time gives us a much more convenient way to utilize the CTCs. We have shown using wormholes a spacetime containing CTCs that can be utilized for hypercomputation by sending only signals back. We have seen that except that the existence of wormholes is less well-supported by experimental evidence than that of rotating black holes, wormhole based hypercomputation has several advantages over the black hole based version.

It would be interesting to create and logically analyze (in the spirit of [4], [5], [22], [37]) an axiomatic theory of relativistic computation containing not only basic concepts required by the spacetime theory but also the concepts needed to formulate the scenario of hypercomputation in the language of the theory. The same way as the axiomatic investigation leads to a deeper (more logical) understanding of relativity theories, it would lead to a better understanding of the theory of relativistic hypercomputation.

We do not claim that the story ends here or that all the questions of wormhole based hypercomputation are answered here. On the contrary, we think that there remain several interesting unanswered questions and our main motivation is to arouse the interest about the subject of

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<sup>10</sup>This result is based on a joint research of Mike Stannett (University of Sheffield) and our research group led by Hajnal Andréka and István Németi (Rényi Institute) [6].

wormhole based hypercomputation as a possible rival of the black hole based one.

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